M.A.T.H. Bowl 2024
Coaches Preparation

Algebraic Thinking Focus

Prepared by Mary Auger, Question Coordinator

Introduction

We've come a long way from a half-day workshop in an auditorium with a live team of students to demonstrate how these contests are conducted.

Included in this power point are some great resources to help students learn wonderful math. Please feel free to email me if I can help in other ways.

Introduction

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Jr High math teacher and Department Chair, and part of M.A.T.H. Bowl as a coach or question writer since the 1980's.

And I'm still a M.A.T.H. fanatic,
with a terrific support team of writers and editors!

Calculators

Some problems have fast time limits where we assume students will pick up a calculator while the problem is still being read.

We advise an algebraic calculator so it works order of operations correctly. 6 + 3 x 5 = 21, not 45.

Calculators that allow fraction input and fraction displays do not confuse students like arithmetic calculators do because they follow the rules.

Standards

The contest is for students enrolled in grades 4-6. It may involve material from **any standard** in grades **K-4** as background.

The standards used from grades **5, 6 & 7** include ***Number Sense, Computation, and a focus on Algebraic Thinking***. Standards are minimum goals, but our students are competing to enrich their academic performance. There may even be topics from higher grade levels or other topics that are narrowed and explained here.

Fair game standard topics

4.M.3

Use the four operations to solve real-world problems involving distances, intervals of time, volumes, masses of objects, and money. Include addition and subtraction problems involving simple fractions and problems that require expressing measurements given in a larger unit in terms of a smaller unit.

4.M.4

Apply the area and perimeter formulas for rectangles to solve real-world problems and other mathematical problems. Recognize area as additive and find the area of complex shapes composed of rectangles by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts; apply this technique to solve real-world problems and other mathematical problems.

4.M.5

Understand that an angle is measured with reference to a circle, with its center at the endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. Understand an angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure other angles. Understand an angle that turns through n one-degree angles is said to have an angle measure of n degrees.

Fair Game – if we narrow it or explain it here

5.DS.2

The mode is easy to find. If it is defined in the problem as a reminder, that makes this data analysis topic fair game in an algebra year, even though it is a data analysis item for grade 5.

Standard: Measurement

3.M.3
Tell and write time to the nearest minute from analog clocks, using a.m. and p.m., and measure time intervals in minutes. Solve real-world problems involving addition and subtraction of time intervals in minutes.

Know 60 seconds = 1 minute

60 minutes = 1 hour

12 months = 1 year

3.M.5
Find the area of a rectangle with whole-number side lengths by modeling with unit squares, and show that the area is the same as would be found by multiplying the side lengths. Identify and draw rectangles with the same perimeter and different areas or with the same area and different perimeters.

3.M.6
Multiply side lengths to find areas of rectangles with whole-number side lengths to solve real-world problems and other mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

Standards

In addition, there will occasionally be material from more advanced grade levels. Where that happens, the topic will be narrowed here in this presentation so you can prepare students adequately. The examples given here will help when that happens. The following screens include topics that are used this year in the Invitational, Area, or both contests.

Advanced Topics

We use a base ten system with ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The place values are:

10^4 10^3 10^2 10^1 $10^0 = 1$

2,356 is 2 x 1,000 + 3 X 100 + 5 x 10 + 6 x 1 =

2,000 + 300 + 50 + 6

Modular Math concentrates on the remainders under division by a fixed positive integer. Given the pattern:

6, 12, 18, 24, 30, 36 ….. What would the units digit be in the 42^{nd} term?

6! (read 6 factorial) = 6 x 5 x 4 x 3 x 2 x 1 = 720

Vocabulary & Topics Used

Order of Operations: parentheses, exponents, multiply and divide left to right, then add and subtract left to right.

Measure of central tendency: **Mode** occurs most often in a data list. **Average** (mean) is sum of items divided by the number of items in a data list.

Greatest common factor can be found by listing all factors, upside down division (division ladder), or prime factorization.

Least common multiple is found by listing the multiples and looking for the smallest one in common.

Vocabulary & Topics Used

Know the **squares** and **cubes** of 1-10 and be able to find any square or cube. 17 squared is 289 and 18 cubed is 5,832. Also, understand that the square root of a square number is itself.

Find the missing or next term in patterns based on **arithmetic and geometric sequences**.

2, 4, 6, ___, 10, 12

2, 4, 8, 16, ?, 64, 128, ?

Parts add up to the whole. A pizza is divided so John gets ½ and Bob gets ½ of what is left. How much does Susie get?

½ + ½(½) + x = 1

Vocabulary & Topics Used

The formula or equation that gives the sum of the interior angles of polygons can be written as **S = 180(n-2)**. So a 102-sided polygon has angles that add to 180(102 – 2) or 18,000 degrees. Although it uses geometry, students need to be able to look for the pattern or use the **formula,** which is an algebra topic.

Consecutive multiples – counting by that number. Three consecutive multiples of 8 might be 16, 24, and 32. Four consecutive multiples of 9 might be 90, 99, 108, and 117. If n= smallest multiple of 5, the next one is n+5, then n+10, …

Vocabulary & Topics Used

A **ratio** compares two numbers and can be written several ways. 1 to 2 , 1:2 , ½ all mean the same comparison.

A **proportion** tells us two ratios are equivalent. We solve these equations by finding the **cross products.**

Understand **percent** as a number compared to 100. Be able to write a percent as fraction or decimal. Solve percent problems.

Simplify expressions with **exponents** such as 7^3 is 7 x 7 x 7 or 343.

Simple Percent Questions

Write 10% as decimal and fraction.

0.10 or 10/100

Find 10% of 345.

0.10(345) = 34.50 multiply then put in decimal point. “Of” means to multiply.

Suggest NOT using % key on calculator as they don’t all work the same way.

Find percent of discount or markup has lots of more difficult examples with full solutions.

Algebra Topics

Write and **solve one step equations** like $x + 5 = 21$.

Undo the operation shown with the opposite operations to both sides of the equation. Solve this equation by subtracting 5 from both sides.

Write and **solve two step equations** like $2x – 9 = 11$.

Undo the addition or subtraction first, then the multiplication or division. This is the opposite order of operations to find value of variable.

Algebra Topics

Simplify expressions by combining like terms:

3 + 9 + 2x simplifies to 2x + 12, as only the constants can be combined.

5x – 4 + 2x + 6 simplifies to 7x +2, as we can combine the to constant terms and the two variable terms.

x + 13 – 9x – 13 simplifies to -8x, because the “+13” and the “-13” become zero, which no longer needs to be written and “x” has an understood 1 coefficient.

Algebra Topics

Translate and solve a word problem into a linear equation that has the variable on both sides of the equals sign.

If 5 is added to twice a number the result is 25 less than three times the number.

Let n = the number

5 + 2n = 3n - 25 results from direct translation of words, where subtraction order may confuse some

Solve by subtracting 2n from both sides first, then adding 5 to both sides, we get n = 30.

Solve linear equations with variables on both sides.

Problem Solving Processes

Check each of the answers given in the word problem. This usually means working backward.

Make a table, which is often what is done before students learn to write equations.

Work backwards.

Write and solve equations.

Problem Solving

Simplify the problem and look for a pattern.
Suppose every 5 minutes, I walk 2 blocks, then go back one block. I then repeat this process every five minutes until I get to school, 12 blocks away. How many times will I walk forward?

Simplify:
The school is 2 blocks away. Will I go back one block? NO! How many times did I walk forward?
Now what if the school is 3 blocks away?

Solving Work Problems

When two people work to do a job, it is usually at different rates. When they work together, it takes less time. Algebra books have full lessons about these problems but here is a great website to cover the questions asked in this year's contest.

Video Example
Text Example

Ratios & Proportions

Sample lessons using ratios and proportions

Have students write the words for each numerator and denominator first. It's hard to get it wrong that way.

The recipe calls for 3 cups of flour and 2 cups of sugar. If I want to use 4 cups of sugar, how much flour will I need?

$$\frac{\text{Flour}}{\text{Sugar}} \quad \frac{3}{2}$$

Now the second fraction must also be flour/sugar.

$$\frac{\text{Flour}}{\text{Sugar}} \quad \frac{3}{2} = \frac{x}{4}$$

Watch for unit changes: mph to seconds per hour

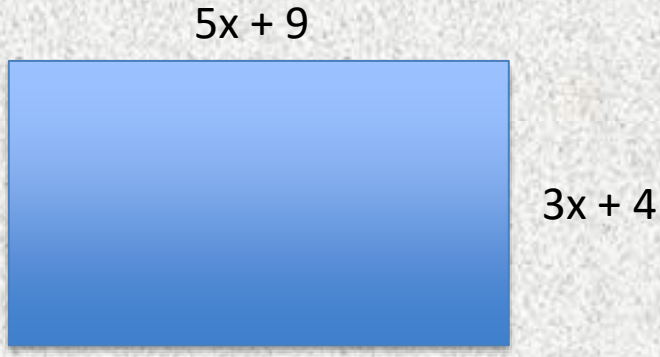
Ratios & Proportions

Worksheets using drawings and maps

Scale drawings and map problems are often solved with a proportion.

Algebra from Picture

The perimeter of a rectangle is the sum of the lengths of the sides. Find the perimeter if x = 10.



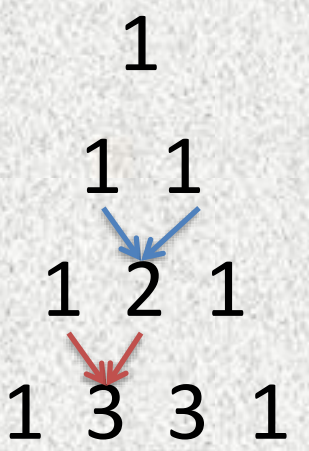
5x + 9 = 5(10) + 9 or 59 length of rectangle

3x + 4 = 3(10) + 4 or 34 width of rectangle

Perimeter = 59 + 34 + 59 + 34 or 186 *(from words in the problem rather than a geometry formula.)*

Pascal's Triangle

Be familiar with the triangle and how each entry is formed by adding the two numbers in the row immediately above.



Each row starts and ends with 1.

First two rows are given.

1 + 1 = 2

1 + 2 = 3

Prime Numbers & Divisibility

Students should be able to list prime factors and find GCF.

The factors of 10 are 1, 2, 5, and 10.

The factors of 25 are 1, 5, and 25.

The common factors are 1 and 5.
The greatest of the common factors (GCF) is 5.

Prime Numbers & Divisibility

Prime numbers have exactly two factors, one and themselves. 1 is neither prime nor composite.

Students should know 2, 3, 5, 7, 11, 13, 17, 19 are prime. Students should also know how to test for divisibility.

Even numbers are divisible by 2.

Numbers whose digits add to a number divisible by three are themselves divisible by three.

Numbers that end in 0 or 5 are divisible by 5.

It may be easier to use a calculator to test for divisibility by 7, 11, 13, 17, 19, ….

2 Equations, 2 Unknowns

In some word problems, there are 2 unknown quantities. In that case, we must write 2 equations using 2 variables.

All of our equations will be **linear** (no exponents on the variables).

All our **systems of 2 equations in 2 unknowns** can be solved with substitution or simple elimination. If using an 8^{th} grade or Algebra 1 book, look only at first couple of lessons for examples in that chapter.

2 Equations, 2 Unknowns - Elimination

There are problems that can be solved using 2 equations with 2 variables. Those can get difficult to solve, but the only methods needed in this contest are those solved with substitution or simple elimination. For elimination, the 2 equations are added or subtracted from one another because one set of coefficients is the same or opposites.

Here's some help.

2 Equations, 2 Unknowns ➔➔ Elimination

$5x + 7y = 31$

$2x - 7y = 4$

Adding these gives us $7x = 35$, then $x = 5$. If we need y, we put the value of x into one equation and solve for y. In this case, it's a fraction 6/7.

2 Equations, 2 Unknowns ➔➔ Substitution

$5x + -3y = 31$

$x = y - 7$

$5(y - 7) + -3y = 31$	Replace x with y-7
$5y - 35 + -3y = 31$	Distribute the 5 times
$2y - 35 = 31$	Subtract 2y from both sides
$2y = 66$	Add 35 to both sides
$y = 33$	Divide by 2

Piles of coins & substitution

A pile of coins consists of only 2 kinds, say nickels and dimes. You are given how many total and their total value.

Let x = number of nickels

Let y = number of dimes

The first equation is x + y = total coins or

y = total number of coins – x

The second equation is based on the value

5x + 10(total coins – x) = total value

Handshake Problems

If six people enter a room and shake hands with every other person, how many handshakes occurred?

Fun to act out (pretend this year!) and record in a table. Can also be solved by drawing a picture or looking for the pattern that results in triangular numbers.

See Owlcation.com for a full explanation.

Heads and Legs problems

Some 2-legged and 4-legged animals are in a zoo, yard, farm, etc. Given the number of heads and legs, find the number of each animal.

Teaching the problem solving and algebra

Then, you might show them the

NO Algebra solution

Sequences

Students may be asked to find the next term or the missing terms in a sequence of numbers.

24, 25, 26, …. are integers

24, 26, 28, … are even integers

35, 37, 39, … are odd integers

7, 14, 21, … are multiples of 7

7, 14, 21, … is also an arithmetic sequence because we add the same value each time

Sequences

2, 4, 8, 16, 32, … is a geometric sequence because we multiply by the same factor each time

1, 4, 9, 16, 25, 36, … is a sequence of squares

1, 8, 27, 64, 125, … is a sequence of cubes

1, 1, 2, 3, 5, 8, 13, … is the Fibonacci sequence. Each term is the sum of the previous 2 terms

3, a, 6, b, 9, c, 12…. is an arithmetic sequence

Consecutive means they follow in a pattern.

Arithmetic Sequences

There is a formula to find any value of terms in a sequence.

The nth term is the sum of the first term plus the common difference multiplied by one less than n.

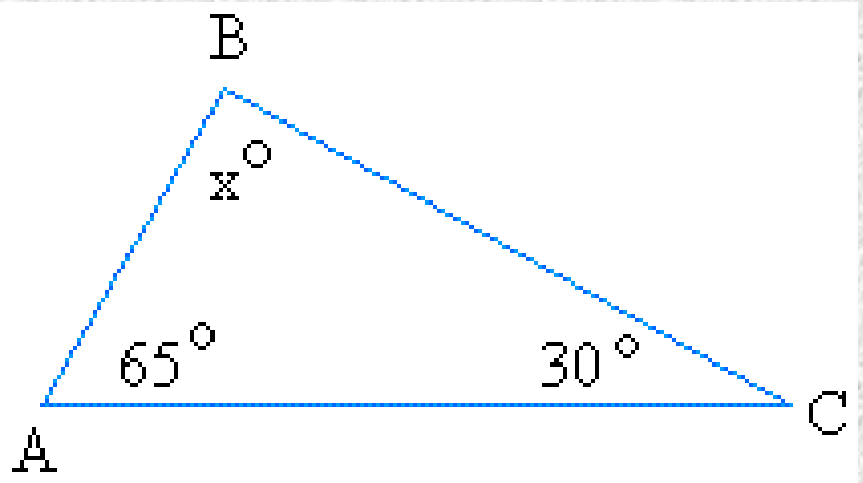
$X_n = a + d(n-1)$

Math Is Fun Explanation of Formula

Sum of the Angles in a Triangle

The three angles in any triangle will add to 180 degrees. This fact will be given this year.

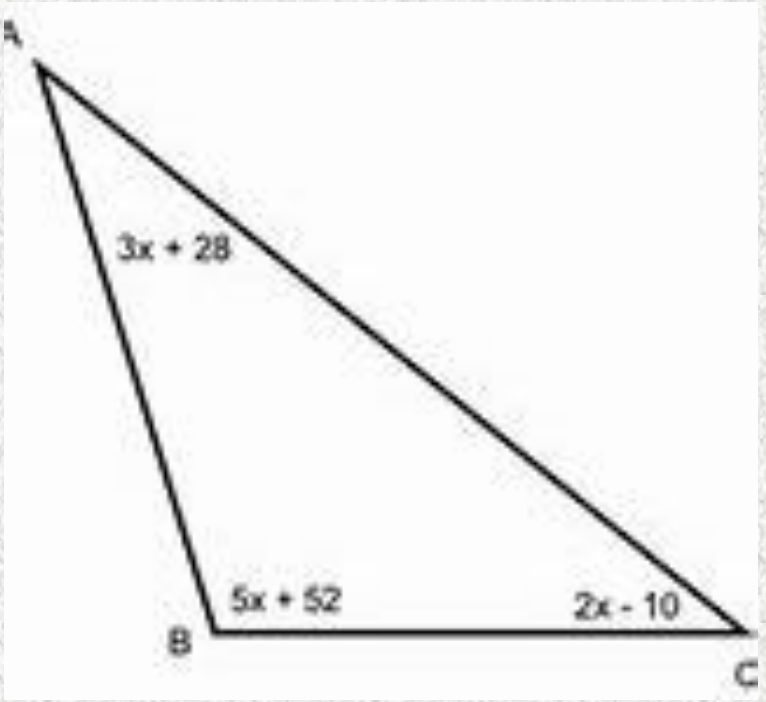
Then, students will be asked to find the value of one of those angles. It is a 30 second question if 2 of the 3 angles are then given.



65 + 30 + x = 180
95 + x = 180
x = 180 – 95
x = 85

Sum of the Angles in a Triangle

The three angles in any triangle will add to 180 degrees. It becomes a higher level question with more time if the angle measures involve variables. Find the measure of the smallest angle.



(3x + 28) + (5x + 52) + (2x – 10) = 180
10x + 70 = 180
10x = 110
x = 11

(3x + 28) = 3(11) + 28 = 61
(5x + 52) = 5(11) + 52 = 107
(2x – 10) = 2(11) – 10 = 12

Try-Angle Sum question

If the angles are in a ratio, it's more fun.

Suppose the angles are in the ratio 1:2:3.

Then we let the smallest angle have measure x. The next angle measures 2x. The third is 3x.

Since they add to 180 degrees, we get

$x + 2x + 3x = 180$

$6x = 180$

$x = 30$

So the three angles have measures 30, 60 and 90 degrees.

ASK-ROSE Homework Hotline

Homework help is available for students in grades 6-12 by calling 877-ASK-ROSE (877-275-7673) Sunday through Thursday from 7-10 p.m. or visiting ASKROSE.ORG for webpage and chat help.

Our topics are middle school topics so these tutors can help!